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Modified renormalon

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Abstract

In the framework of renormalon consideration in the approximation of a large number of quark flavours, a role of the anomalous dependence of gluon propagator on the scale μ is shown.

In the QCD perturbation theory, an account for the next-to-leading order term over α_s is connected with the uncertainty, caused by a choice of the energy scale, determining the $\alpha_s(\mu^2)$ value, since the transition to the $\bar{\mu}$ scale leads to the substitution

$$\alpha_s(\mu^2) \approx \alpha_s(\bar{\mu}^2)\Big(1 - \alpha_s(\bar{\mu}^2)\frac{\beta_0}{4\pi}\ln\frac{\mu^2}{\bar{\mu}^2}\Big)\ ,$$

where $\beta_0 = (11N - 2n_f)/3$, N is the number of colours, n_f is the number of quark flavours. So, the physical quantity, represented in the form

$$r = r_0\alpha_s(\mu^2)\{1 + r_1\alpha_s + O(\alpha_s^2)\}$$

with the given order of accuracy, will get the form

$$\begin{aligned} r &= r_0\alpha_s(\bar{\mu}^2)\Big\{1 + \Big(r_1 - \frac{\beta_0}{4\pi}\ln\frac{\mu^2}{\bar{\mu}^2}\Big)\alpha_s + O(\alpha_s^2)\Big\} \\ &= r_0\alpha_s(\bar{\mu}^2)\{1 + \bar{r}_1\alpha_s + O(\alpha_s^2)\}\ . \end{aligned}$$

Thus, the value of r_1 coefficient for the α_s correction depends on the scale of $\alpha_s(\mu^2)$ determination. In ref.[1], one offered to fix the μ scale so that the r_1 coefficient does not contain terms, proportional to β_0. The latter procedure can be understood as one, leading to r_1 to be independent of the number of quark flavours $n_f \sim \beta_0$. As was shown in refs.[2, 3], such choice of scale has the strict sense in the framework of the $1/n_f$ expansion, where the α_s correction to the gluon propagator is determined by the fermion

loop contribution into the vacuum polarization. So, the expression for such contribution depends on the regularization scheme, and in the next-to-leading order one has

$$\alpha_s(\mu^2)D(k^2,\mu) = \frac{\alpha_s(\mu^2)}{k^2}\left\{1+\frac{\alpha_s n_f}{6\pi}\left(\ln\frac{k^2}{\mu^2}+C\right)\right\}, \tag{1}$$

where $D(k^2,\mu)$ determines the transversal part of the gluon propagator $D^{ab}_{\nu\lambda}(k^2,\mu) = \delta^{ab}D(k^2,\mu)(-g_{\nu\lambda}+k_\nu k_\lambda/k^2)$. The constant value C is defined by the renormalization scheme, so $C_{\overline{MS}} = -5/3$ and $C_V = 0$ in the so-called V-scheme [1].

The summing of $(n_f\alpha_s)^n$ contributions into the gluon propagator leads to the expression

$$\alpha_s(\mu^2)D(k^2,\mu) = \frac{\alpha_s(e^C k^2)}{k^2}, \tag{2}$$

i.e., in fact, it results in the account for the "running" α_s value in respect to the gluon virtuality. Note, the $1/n_f$ consideration is exact for the abelian theory, where $\alpha_s D$ is the renormalization group invariant. Moreover, the $e^{-C}\Lambda^2_{QCD}$ value does not depend on the renormalization scheme, and therefore it results in the scheme-independence for the expression

$$\alpha_s(e^C k^2) = \frac{4\pi}{\beta_0 \ln(e^C k^2/\Lambda^2_{QCD})}.$$

As one can see in the performed consideration, the transition to the nonabelian theory is done by the $2n_f/3 \to -\beta_0$ substitution, that is called as the procedure of "naive nonabelization", giving the correct "running" of the coupling constant.

Further, consider the physical quantity r, for which the first order α_s-contribution is calculated as an integral over the gluon virtuality with the weight $F(k^2)$

$$r = \int \frac{dk^2}{k^2}\,\alpha_s(\mu^2)F(k^2).$$

Then the account for the $(n_f\alpha_s)^n$ corrections to the gluon propagator leads to the substitution of "running" value $\alpha_s(e^C k^2)$ for $\alpha_s(\mu^2)$

$$\alpha_s(\mu^2) \to \alpha_s(\mu^2)\sum_{n=0}^{\infty}\left\{-\frac{\alpha_s(\mu^2)\beta_0}{4\pi}\left(\ln\frac{k^2}{\mu^2}+C\right)\right\}^n, \tag{3}$$

so that in the offered procedure of the scale fixing [1], the introduction of the first order $n_f\alpha_s$-correction results in the substitution $\mu^2 \to \bar{\mu}^2 = \mu^2\exp\{C+\langle\ln k^2/\mu^2\rangle\}$, where the average value is determined by the integral with the F weight. However, in some cases [2, 3] the integration of the n-th order term in expansion (3) leads to the $n!$ factorial growth of the coefficients for the expansion over $\alpha_s^n(\mu^2)$, so that this rising can be determined by the region of low virtualities as well as large ones of gluon. Then one

says about the infrared and ultraviolet renormalons, respectively [2, 3, 4]. As was shown [2, 3], the renormalon leads to power-like uncertainties of the r evaluation, so

$$\Delta r = \left(\frac{\Lambda_{QCD}}{\mu}\right)^k a_k \;, \tag{4}$$

where k is positive for the infrared renormalon and it is negative for the ultraviolet one. For the two-point correlator of heavy quark vector currents, for instance, one has $k = 4$ and the corresponding uncertainty can be, in fact, eliminated in the procedure of definition for the nonperturbative gluon condensate, having the same power over the infrared parameter [3]. However, for the renormalized mass of heavy quark the infrared renormalon with $k = 1$ does not correspond to some definite quark-gluon condensate, and the value $\Delta m(\mu) \sim \Lambda_{QCD}$ can not be adopted into a definition of a physical condensate.

In the present paper we modify the renormalon due to the account for the anomalous dependence of gluon propagator on the μ scale, and the modification turns out, basically, to be essential for numerical estimates, related with the determination of physical scale, fixing α_s.

In the covariant gauge, the gluon propagator has the form

$$D^{ab}_{\nu\lambda}(k^2,\mu) = \frac{\delta^{ab}}{k^2\omega(\mu^2,k^2)}\left(-g_{\nu\lambda} + (1-a_l(\mu^2)\omega(\mu^2,k^2))\frac{k_\nu k_\lambda}{k^2}\right) \;,$$

where $a_l\omega$ is the renormalization group invariant. The solution of one-loop equation of the renormalization group for $a_l(\mu^2)$ in the $\overline{MS}$-scheme allows one to represent the gluon propagator in the form [5]

$$D^{ab}_{\nu\lambda}(k^2,\mu) = \frac{\delta^{ab}}{k^2}\frac{1}{1-(1-\omega(\mu_0^2))\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{n_a}}\left(-g_{\nu\lambda} + (1-b)\frac{k_\nu k_\lambda}{k^2}\right) \;, \tag{5}$$

where $n_a = (13N - 4n_f)/(6\beta_0)$ and b is the arbitrary gauge parameter[1]. In eq.(5) we take into account the anomalous dependence for the gluon propagator on the scale, $\omega = \omega(\mu^2)$, only.

The choice of fixed point $\omega \equiv 1$ corresponds to the standard renormalon. The consideration of the gluon propagator at $\omega \neq 1$ leads to the modified renormalon.

In the leading order over $1/n_f$ one has $n_a = 1$, and the cases with 1) $\omega(\mu_0^2) > 1$, 2) $\omega(\mu_0^2) = 1$ and 3) $\omega(\mu_0^2) < 1$ at some large μ_0 values can be formally come to the different choices of the renormalization group invariant μ_g, such that $\omega(\mu_g^2) = 0$ and 1) $\mu_g < \Lambda_{QCD}$, 2) $\mu_g = \Lambda_{QCD}$ and 3) $\mu_g > \Lambda_{QCD}$, respectively. The $\alpha_s(\mu_g^2)$ value is considered as the formal one-loop expression

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln\frac{\mu^2}{\Lambda^2_{QCD}}} \;.$$

[1] In ref.[5] the solution with the fixed $b = 3$ value is considered and ω is expressed through b/a_l.

Thus, the $1/n_f$ consideration gives

$$\frac{\alpha_s(\mu^2)}{\omega(\mu^2)} = \frac{\alpha_s(\mu^2)\alpha_s(\mu_g^2)}{\alpha_s(\mu_g^2) - \alpha_s(\mu^2)} = \alpha_s(e^d\mu^2) \; , \tag{6}$$

where d is the scheme-independent invariant of the one-loop renormalization group, and it is equal to

$$d = \ln\frac{\Lambda_{QCD}^2}{\mu_g^2} = -\frac{4\pi}{\beta_0\alpha_s(\mu_g^2)} \; .$$

Further, one can repeat the calculation of $(n_f\alpha_s)^n$ contributions into the gluon propagator with the substitution of value (6) for $\alpha_s(\mu^2)$, so that this replacement corresponds to the account for the anomalous dependence of the gluon propagator versus the μ scale. Then the scale fixing due to the next-to-leading order α_s correction results in the expression

$$\bar{\mu}^2 = \mu^2 \exp\{C + \langle \ln k^2/\mu^2\rangle + d\} \; ,$$

and the summing of the corresponding contributions modifies eq.(2)

$$\alpha_s(\mu^2)D(k^2,\mu) = \frac{\alpha_s(e^{C+d}k^2)}{k^2} = \frac{\alpha_s(e^C k^2)}{\omega(e^C k^2)k^2} \; . \tag{7}$$

Next, one can define the $\overline{V}$-scheme, introducing $C_{\overline{V}} = -d$. Then

$$\Lambda_{QCD}^{\overline{V}} = e^{5/6-d/2}\Lambda_{QCD}^{\overline{MS}} \; .$$

In the $\overline{V}$-scheme the perturbative potential between the heavy quark and antiquark in the colour-singlet state will have the form

$$V(q^2) = -\frac{4}{3}\,\frac{4\pi\alpha_s^{\overline{V}}(q^2)}{q^2} \tag{8}$$

at $q^2 = \mathbf{k}^2$. Potential (8) comes to the Richardson potential [6], when one uses $\alpha_s^{\overline{V}}(q^2 + \Lambda_{QCD}^2)$ instead of $\alpha_s^{\overline{V}}(q^2)$, and Λ_{QCD} fixes the linearly rising part of potential, confining quarks with the distance increase,

$$\Delta V_{lin}(\mathbf{x}) = \frac{8\pi\Lambda_{QCD}^2}{27}|\mathbf{x}| \; .$$

The fitting of mass spectra for the charmonium and bottomonium in the Richardson potential gives

$$\Lambda_{QCD}^{\overline{V}} = 400 \pm 15 \text{ MeV} \; .$$

The one-loop expression with $\alpha_s^{\overline{MS}}(m_Z^2) = 0.117 \pm 0.005$ [7] corresponds to $\Lambda_{QCD}^{\overline{MS}} = 85 \pm 25$ MeV, that allows one phenomenologically to determine the values

$$\begin{aligned} e^{-d/2} &= 2.1 \pm 0.5 \;, \\ \mu_g^{\overline{MS}} &= 180 \pm 45 \text{ MeV} \;, \\ \alpha_s(\mu_g^2) &= 0.94 \pm 0.21 \;. \end{aligned} \tag{9}$$

However, an additional uncertainty into the estimates is involved by the transition from the number of flavours $n_f(m_Z) = 5$ to another value $n_f(1 \text{ GeV}) = 3$. Obtaining eq.(9), one has supposed, that Λ_{QCD} does not depend on the number of flavours, so that at the scale of "switching on (off)" the additional flavour of quarks $\mu = m_{n_f+1}$, the QCD coupling has a discontinuity, related with the step-like change $\beta_0(n_f) \to \beta_0(n_f+1) = \beta_0(n_f) - 2/3$. One can avoid such discontinuities, if one supposes that Λ_{QCD} depends on the flavour number, so that $\alpha_s(\mu^2 = m_{n_f+1}^2, \Lambda_{QCD}^{(n_f)}, n_f) = \alpha_s(\mu^2 = m_{n_f+1}^2, \Lambda_{QCD}^{(n_f+1)}, n_f+1)$. Then in the one-loop approximation one finds

$$\Lambda_{QCD}^{(n_f)} = \Lambda_{QCD}^{(n_f+1)} \left(\frac{m_{n_f+1}}{\Lambda_{QCD}^{(n_f+1)}} \right)^{\frac{2}{3\beta_0(n_f)}} \;. \tag{10}$$

Setting $\Lambda_{QCD}^{(5)} = 85 \pm 25$ MeV, one gets

$$e^{-d/2} = 1.24 \pm 0.32 \;. \tag{11}$$

Hence, the α_s rescaling, accounting for eq.(10), results in the d value, that agrees with the standard renormalon ($d \equiv 0$) within the current accuracy.

Next, one can use the nonabelian "running" of ω with $n_a \neq 1$ and the same scale as in the "running" of α_s. In the latter procedure one has two scales Λ_{QCD} and μ_g, so that two "measurements" of α_s/ω quantity, entering physical values, are needed. If one fixes μ_g^V from the linear part of the heavy quark potential and uses the Z pole data for the Λ_{QCD} evaluation, then the obtained values show the standard renormalon validity within errors.

Thus, we have found that in the $1/n_f$ consideration the account for the anomalous dependence of the gluon propagator on the μ scale results in the modification of renormalon because of the presence of μ_g scale in addition to Λ_{QCD}, and the difference from the standard renormalon turns out to be essential in numerical estimates. We have also determined $\mu_g^{\overline{MS}}$ in a phenomenological manner.

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